Young children's use of drawings in addition problems

Uso de dibujos por parte de niños pequeños en problemas de suma

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ABSTRACT

Understanding addition concept is troublesome for many young children. For individuals who struggle in comprehending this abstract concept, alternative representation form may help address such difficulty. This paper explores the drawings created by young children (6 years old) in addition problem solving activities. The study employed case study research design involving six children (aged six years) in three preschool centres. Data collection included observation, informal interviews and analysis of drawings. The findings showed that young children created two types of drawing and that the processes involved in producing the visual representations had facilitated the children's understanding of addition. The study implicated that young children's creations of drawings is an important learning experiences and could be best assisted by valuing and supporting the early development of children's drawn mathematical representation.

Keywords: Young children, Addition, Problem Solving, Representation, Drawing.

RESUMEN

Comprender el concepto de suma es problemático para muchos niños pequeños. Para las personas que luchan por comprender este concepto abstracto, la forma de representación alternativa puede ayudar a abordar dicha dificultad. Este artículo explora los dibujos creados por niños pequeños (6 años) además de actividades para resolver problemas. El estudio empleó un diseño de investigación de estudio de caso que involucró a seis niños (de seis años) en tres centros preescolares. La recolección de datos incluyó observación, entrevistas informales y análisis de dibujos. Los hallazgos mostraron que los niños pequeños crearon dos tipos de dibujos y que los procesos involucrados en la producción de las representaciones visuales habían facilitado la comprensión de la suma de los niños. El estudio implicaba que las creaciones de dibujos de los niños pequeños son experiencias de aprendizaje importantes y podrían ser mejor asistidas valorando y apoyando el desarrollo temprano de la representación matemática dibujada de los niños.

Palabras clave: Niños pequeños, Suma, Resolución de problemas, Representación, Dibujo.

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1. INTRODUCTION

Developing students' understanding is highlighted as an important objective of mathematics education (Hiebert, 1997). The National Council of Teachers of Mathematics (NCTM, 2000) emphasize that mathematics should be learnt with understanding. This skill is vital in the early years of school where young children are in the process of developing basic mathematics skills. Also it is critical that teachers' knowledge and skills as well as current practice help prepare student with the capability of competing and facing challenges in this global world by practicing the 21st teaching and learning approaches (Ali & Maat, 2019; Bakar, Maat, Rosli, 2019). Representation created and utilised by students that aligns with the constructivist theory of learning are beneficial in learning various mathematics concepts.

Glasersfeld (1995) asserts that an individual understanding of a concept could not observed directly. However, a student's understanding of a concept can be inferred through the representations created by him/her. Hence, "representations become crucial to our understanding of how students grow in their mathematical ideas; serving as mediator in students' growth of understanding and as a means of communicating that understanding to others" (Wilson & Stein, 2007, p. 673).

Research provides extensive evidence relating to the positive influence of representation usage in the teaching and learning of mathematics. Representations has the power to support the communication and sharing of mathematical thinking and ideas, and develop conceptual understanding (Bakar & Karim, 2019; Bakar, 2018; Ainsworth, 1999; Rosli, Goldsby & Capraro, 2015; Ayub, Ghazali, & Othman, 2013). The National Council of Teachers of Mathematics supports the use of representation in both instruction as well as learning to facilitate the understanding of abstract concepts (NCTM, 2000). As gaining understanding is an important goal in mathematics (Hiebert, 1997), representation can play an important function as there exist strong relationships between representation and mathematical understanding (Yuanita, Zulnaidi, & Zakaria, 2018; Abdullah, Halim & Zakaria, 2014; Abdullah, Zakaria & Halim, 2012; Ainsworth, 1999). Representation of various forms can aid the learning and understanding of mathematics learning by supporting a student's ability to explore, access, justify, reflect, analyze and connect representations (NCTM, 2000). While previous studies have reported the positive function of various types of representations for mathematical learning, children's drawing in early year's mathematics has not been fully explored (Crespo & Kyriakides, 2007: Woleck, 2001) particularly with regards to problem solving. Also, researchers highlight the power of drawings as means for expressing meaning, emotions and experiences (Papandreou, 2014) with little attention to the role of drawings for concept-building.

2. RESEARCH BACKGROUND

R Research highlight the important function of visualization in teaching and learning mathematics. "We could not even imagine introducing many mathematical concepts without illustrating them by pictures, drawings, graphs, etc." (Zarzycki, 2004, p. 108), especially to young children who rely more on visual than adults. Researchers indicate the important role that visualization plays in problem-solving (Edens & Potter, 2007; Fagnant & Vlassis, 2013; Zahner & Corter, 2010). Rösken and Rolka (2006) found that students used visualization creatively and modified the tasks to help them to work on problems. Visualization involves conversion from external to mental images (or vice versa) and, particularly, the link between the physical image and the mental image made by the individual. Making sense of various mathematical concepts can be assisted by visualization. The use of images as exist in mathematical picture books, activities and tasks are helpful in building students' comprehension of mathematics concepts.

Drawings as one form of visual representations play an important function in problem-solving (Edens & Potter, 2007; Fagnant & Vlassis, 2013; van Garderen, 2007). They are useful in helping students' understanding of the mathematical problem, they facilitate students to build the vital mathematical concepts necessary to arrive at the solution (Abdullah, Zakaria, & Halim, 2012) and hence, to solve the problem successfully (Bakar, 2017; Stylianou, 2010; van Garderen, Scheuermann, & Jackson, 2013). One of the factors prompting students to utilize visual representation is its' benefits in reducing the level of cognitive load and, thus, reducing problem difficulty (Cankoy & Özder, 2011). Further, when students use accurate schematic representations, their chances of solving problems successfully increased (Boonen, van Wesel, Jolles, & van der Schoot, 2014). Furthermore, drawings help to demonstrate to others how they approached the problems (Zahner & Corter, 2010).

Despite the fact that drawings facilitated problem solving processes, there is no guarantee that children's self-generated drawings automatically linked to problem solving performance. The lack the mathematical knowledge required for the problem solution (Essen & Hamaker, 1990) affected the resolution of the problem. Additionally, there is no guarantee that a correct resulted in a correct solution (Crespo & Kyriakides, 2007). Researchers comparing the different types of drawings produced by children and the link drawings had with children's success in solving problems found the importance of producing schematic drawings as this type of drawings facilitated children's problem solving performance (Eden & Potter, 2007).

However, researchers highlight their concern pertaining to the status that drawings had for teaching and learning mathematics. In teaching and learning mathematics, drawing is less valued in many grade levels (Soundy & Drucker, 2009)) in comparison to other mathematical representation forms. Drawings are perceived as not useful for mathematical purposes (Essen & Hamaker, 1990) hence resulted in the low usage of this particular representation form.

3. THE STUDY

This study explores children's creation and use of representations to support mathematics learning, understanding and problem-solving. More specifically, the research seeks to ascertain the ways in which children create and use visual representations to attain concept understanding pertaining to addition and later solve given problem. By taking a closer look into their act, talk and behaviours during the production of drawings whilst finding the solution to posed problems, the children's understanding of the addition concept could be observed.

In particular, this study will address the following research questions:

1. What types of drawings did the children produce?

2. How did children use their drawings in addition problems?

Following the constructivist theory of learning, children were introduced to visual representations and were exposed to creating and using drawings in problem solving. Having informed by the teachers that the children lack experience in creating drawings in mathematics, the researcher prompted the children to create any drawings that were easy to produce that can help solve posed problem. The researcher did not specifically teach the children what and how to draw, rather prompted the children to produce drawings that made sense to themselves. In order to encourage children to create the drawings themselves, the researcher frequently provided assurance that the children's drawings were alright as long as the drawings are meaningful to them. By prompting the children to represent their own meaning of addition in their attempt to solve given problems, they were engaged in exploring and creating their own understanding rather than passively receiving knowledge from the researcher. Since internal representations are linked to external representations (Goldin & Shteingold, 2001), hence internal representations could be inferred through their externalization – as portrayed by the children in this study, in the form of drawing.

The study took place in three 'pre-schools', two in Sarawak and one in Sabah, Malaysia. Six children (from each school) were selected as the participants for this study. In Malaysia, the term 'preschool education' is defined by the Education Act 1996 as educational programs for children of four to six years old and that these children are called 'pre-school children'. The teachers who also played the role of researchers collected data from only this group of children throughout the study, although there were times in which the whole classroom were included during the teaching and learning processes.

At the administration of this study, the children had not yet being given any formal instruction on the concept of addition. They had not been taught how to solve addition problems. The teachers introduced the addition process through modelling with various representation forms including concrete materials, drawings and symbols. After having observed that the children were able to work independently on a number of addition problems, the final tasks were posed. Children were requested to create any representations (including drawing) that would aid them solve the problems verbalized by the researchers. The data for this paper is mainly focused on the drawings created by the children, despite the fact that they may also use other representation forms to find the answer to the same problem. The problems required the children to find the total number of wheels for a number of vehicles. (Problem A: 2 cars and a motorcycle; Problem B: 2 motorcycles and a car and Problem C: A car and a motorcycle)

4. DATA SOURCES AND ANALYSIS

Data were obtained from observations of children completing the tasks, conversations with children, artefacts (children's drawing), field notes and video recording. Firstly, children's drawing were analysed and categorized into different types of drawing. Video analysis of children working out the tasks demonstrated different ways of using drawings whilst attempting the problem solution. Children's talk helped to clarify what they are doing and further explain their thinking involved to complete the tasks. The data were organised by compiling a table annotating each child's drawings and accompanying talk, together with their behaviours that provided insight into their mathematical thinking. Using such tables allowed a child's drawing to be linked across various data sources and thus afford a rich information about each child's drawings. Then, the drawings and thinking of each child were compared. The drawing artefacts, together with the processes involved (identified through observations and conversations with children) provided further information about the function of drawing for problem solving.

5. FINDINGS

Children in this study were prompted to create drawings (an alternative form of visual representation) in addition to other representation forms produced by the children to help solve addition problems. Having produced similar drawings, only the data from Geneva, Abraham, Dania and Farina are presented in this paper, with their responses considered to be representative of participants from all the selected participants. Through the problem solving tasks, the children's perspectives of number and addition were explored.

What types of drawings did the children produce?

In their attempt to solve posed addition problems, the children produced drawings that can be categorized into two types of drawing – *pictographic* and *iconic*. A *pictographic* drawing contains realistic depictions of the intended objects. On the other hand, *iconic* drawing comprises of only simple lines, marks and shapes to exemplify the objects mentioned in the problems. Figures 1, 2 and 3 are examples of both types of drawing created by the children.

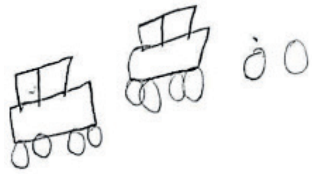

Figure 1: Geneva's drawing: Pictographic

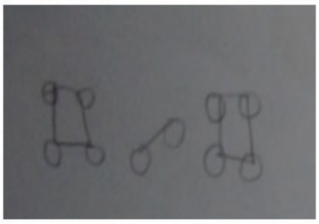

Figure 2: Abraham's drawing: Iconic – (using simple lines and shapes)

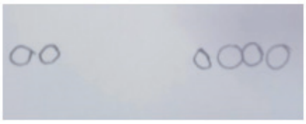

Figure 3: Lisha's drawing: Iconic – (using circles)

In her attempt to solve Problem A, Geneva created a *pictographic* drawing (refer Figure 1). Geneva produced the cars that depicts the vehicle she saw in real life (i.e. with windows and its' body). She perhaps chose not draw the motorcycle as she found it difficult to produce this particular type of vehicle. She then drew the quantity of wheels for a motorcycle. Given the same problem, Abraham instead produced an *iconic* drawing to find the total number of wheels for both vehicles. As seen in Figure 2, Abraham drew a square shape to represent the body of the car and attached four wheels to it. So as to represent a motorcycle, he drew a simple and short straight line as the body of the vehicle and later attached two circles to embody the wheels for the motorcycle. In finding the answer to Problem C, Lisha simply drew two circles to represent the wheels for a motorcycle and included four more circles to embody the wheel for a car (refer Figure 3).

Further analysis of the children's drawings revealed additional subcategories of drawings. Given Problem B, Dania and Farina each produced pictographic and iconic drawings, but these two children included symbols to the drawing they made thus producing an additional two types of drawings- *Pictographic with symbols* and *Iconic with symbols* (refer Figure 4 and 5).

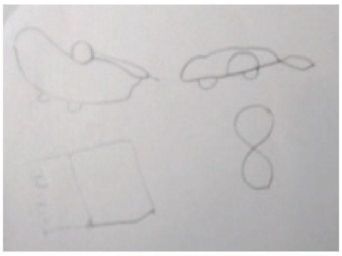

Figure 4: Dania's drawing: Pictographic with symbols

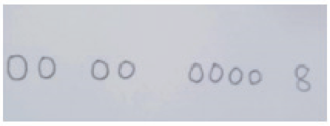

Figure 5: Farina's drawing: Iconic with symbols

Dania produced a *pictographic* drawing (Figure 4) - a picture of two motorcycles and a car that depicts the vehicle in everyday life. She then included the symbol '8' in her drawing as the total number of wheels after counting the marks she made. In contrast, Farina produced an *Iconic* drawing by creating circles to illustrate each wheel (refer Figure 5). Note that no additional details that were not directly relate to the mathematics of the problem question were added to her drawing. She then inserted the numeral '8' as the total wheels for both vehicles that she counted.

How did children use their drawings in addition problems?

Majority of the children created drawings when starting work on the problem. After representing all the wheels for different types of vehicles, Lisha counted them one by one "1,2" (i.e. counting the wheels of the motorcycle) and continued counting the wheels of the car "3,4,5,6". She answered "6" when asked for the total wheels for a motorcycle and a car. Geneva too pointed to her drawings and simultaneously counting them one by one starting from "1" until "10". When asked the total number of wheels, she confirmed that the answer was "10" wheels altogether. For both children, the external representation (drawing of wheels) was important to enable them to apply their counting strategy. Therefore, both Lisha's and Geneva's drawings were integral to their problem-solving strategy.

In contrast, Farina made her drawings after solving the problem using other means. She created her drawings to exhibit other means of representations that can be used to obtain the answer. Initially, Farina choose readily drawn pictures of both vehicles (i.e. flashcards) to help her attempt the problem. She then counted all the wheels (i.e. pictures of the vehicles prepared by the teacher) to help her solve the problem. Upon requested by the teacher, she then recreated the solution in the form of a drawing (Figure 5). By doing so, she made a '*translation*' within the same form of representation (i.e. picture) - from pictures on cards to drawings on paper.

While a number of children verbalized the totals they obtained after making contact with the marks they made, both Dania and Farina rather wrote symbols (in the form of numerals) to record the totals (see Figure 4 and 5). In their drawings, the children represented the quantities for the addends accurately. However, it was not made explicit in the children's drawings that addition comprises combining two groups of objects as there were no additional mark to indicate the act of putting the two groups together. In fact, there were drawings that clearly showed the groups as separated from each other (Figure 3 and 5) as seen by the gaps made between the groups of objects. Despite the absence of any marks to embody the combining of the groups together (Figure 1,2 and 3) and the presence of drawings containing several groups/sets of objects (proved through separation of the groups as seen in Figure 3 and 5), still the children's understanding of the addition concept was made implicit through their actions on the drawings, in which they simultaneously pointed and counted all the marks to get the total.

6. DISCUSSION OF FINDINGS

The findings from the data indicated that the young children's use of drawings provided insights into their understanding of numbers and addition concepts. The creation of drawing during the problem solving processes was found to facilitate the children's understanding of numbers and addition.

This study support the claim made by previous researchers that considers drawing as a meaning-making activity (Papandreou, 2014). In particular relation to the mathematics concepts, this study found drawing as a concept-building activity as there exist relationship between thought and drawing. This is due to the permanent characters of the drawings itself (Brooks, 2009). After the marks are made on the paper, they remain steadily on the same place in which permitted the children to re-examine and assess their process of thinking, talk about it with others, share thinking and ideas (Hopperstad, 2008; Rinaldi, 2001) and act on it (i.e. pointed and counted them as in the case of this study).

The context of the problem (i.e. vehicle problem) plays an important role and had influence on the children's attempt to make drawings. The children's previous experiences (i.e. what a motorcycle or a car looks like) and knowledge (i.e. the number of wheels for the different vehicle) enabled them to create the drawing successfully. Also, knowing such information and having familiar with the vehicles had encouraged the children to make various efforts to overcome the difficulty of creating the vehicles. The children successfully drew the most critical part of the vehicles (i.e. the wheels) required for the solution of the problem, although there were children who included the body of the vehicles. By creating the vehicles in various forms, including drawing 'real' cars and motorcycles or simple lines and shapes, the children successfully demonstrated the ability to represent the *quantities* of the wheels that is vital for solving the problems. There are also children who substituted drawings with numerals to overcome the trouble of producing a large number of drawings for the totals as they found it less troublesome and indeed faster to do so. Furthermore, the inclusion of numerals enabled meanings were communicated clearly. As can be seen in Figure 3 and 4, both the children wrote the numeral "8" to exhibit the total; also to emphasize "8" as the answer/ solution to the problem.

The different marks, lines and shapes produced to represent the problem information revealed the children's different means for making sense of the problem context. It is important to note that regardless of the type of drawing (pictographic or iconic) produced, both type of drawing contributed to the success of the problem solution. In contrast to the findings in the study conducted by Edens and Potter (2007, problem-solution in this study is independent on the quality as well as the detail presented in the drawing. This may be account to the lesser complexity of the problems posed in this study compared to the those problems given to the participants in Edens and Potter (2007) study.

The different means of using drawing exhibited the distinct roles that drawing afforded in problem solving. The students utilised drawing as a means to communicate what they had in mind. Interestingly, the communication of the mathematical thinking was made evident in various phases of the problem solving that served different purposes. Children like Lisha and Geneva relied on the pictures they created to help process the mathematics. As reported by other researchers, the drawing initially functions as a modelling tool and later serve as a problem solving tool to help arrive at the answer (Badillo, Font & Edo, 2014) Clearly, all children profited from the creation of the drawing at all phases during the problem solving processes including the beginning of tasks. However, there are children who produced drawings after finding the solution by utilising other means, such as through modelling with pictures from flashcards. They drew to exhibit to others how they arrive at the solution. Thus their drawing was a *translation* within the same representation mode, for the purpose of confirming the answers. As asserted by Lesh et. al. (1987), facility in making such translations demonstrated children's understanding of mathematical concepts.

The findings from this study generally support the benefits of utilising drawing in mathematical learning and problem solving informed by other studies (Uesaka, Manalo, & Ichikawa, 2007. Furthermore, drawings help open paths for solving difficult problems (Soundy & Drucker, 2009) as happened to the children in this study. Without creating the drawings, it is impossible for the children to find the answer easily; particularly due to the fact that they had not been taught how to solve non-routine problems, and this type of problem do not have an immediate apparent strategy and requires multiple steps for solving them.

7. CONCLUSION, IMPLICATIONS AND RECOMMENDATIONS

This study explored how drawings assist young children acquire understanding of addition concept. The study found that young children produced drawings with different levels of sophistication. Additionally, drawings were produced at various stages throughout the problem solving process. The different role that drawing functioned in this study provided evident for the benefits of drawing - both as a modelling tool as well as a problem-solving tool for young children. The findings from this study support the notion that the transition to drawing as a mathematical representation among children in their early age is non-automatic – indeed not a natural one, hence necessitate effective pedagogical activity from the teacher. Thus, children should experience rich opportunities to create and reflect on different samples of drawn representation, and to clarify and justify their creations. This in turn may help children to further build on various representational strategies for mathematics learning and problem solving.

The findings from this study showing students' ability of making sense of addition concept and solving problems through creating drawings has implications for both the instructions and assessment used in mathematics classroom. When instructions and assessment appreciated only a particular representation form (e.g., symbols), children who are not yet ready to transit to the abstract level (that required other representation forms) would be in disadvantages. Teachers should attend to children's differences in skills, knowledge and learning preferences by promoting the use of multiple representations in classrooms.

It is also suggested that this study will inspire teachers to inspect their present approaches and practices specifically concerning the use of representations in mathematics classrooms. In particular relation to visual representation use, drawings as an alternative modes of representations should be valued in school setting. Both teachers and students should appreciate the power of drawings, as this type of representation is beneficial in helping teachers to introduce new mathematics concepts, also useful in assisting students to comprehend and grasp the concept and later solve mathematical tasks successfully.

As teachers aid young children's transition into abstraction (the world of symbol systems), it is vital that educators warrant that various alternative representation forms (e.g. drawings, talking) are afforded equal emphasis, value and importance. Young children should be prepared for a world of visually oriented modes of technological learning, and their early numeracy educators should accept visual responses as valid intellectual expressions and communication of meaning. Both teachers and students should value drawing as a unique mode of learning for expressive and problem solving purposes.

Further research, including longitudinal study, is required to determine effective pedagogies as well as learning experiences that will develop young children's perception, confidence as well as competence in employing drawing as a learning means as well as problem-solving tool. Such research should include the focus on the whole problem-solving processes, in addition to inspecting 'drawing' as a completed product.

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