Integration of Spatial Visualization Tasks to Enhance Students' Levels of Geometric Thinking following the Van Hiele Model: A Basis for the Development of a Definitive Guide in Geometry

Integración de Tareas de Visualización Espacial para Mejorar los Niveles de Pensamiento Geométrico de los Estudiantes siguiendo el Modelo de Van Hiele: Una Base para el Desarrollo de una Guía Definitiva en Geometría

Robin B. Dimla*
Don Honorio Ventura Technological State University - PHILIPPINES
rbdimla319@dhvtsu.edu.ph

Dynah M. Soriano*
Don Honorio Ventura Technological State University - PHILIPPINES
dynsor84@gmail.com

ABSTRACT

Students who have the ability to manipulate shapes in free play situations, such as building, solving spatial problems, drawing two and three-dimensional objects, exploring shapes through physical actions, describing shapes from different perspectives, and fitting shapes together are commonly observed to have a more advanced level of Geometric thinking. From this perspective students' level of geometric thinking is associated with spatial visualization ability. Hence, the study was conducted specifically to develop a definitive guide integrating spatial visualization tasks to enhance students' level of geometric thinking following the van Hiele model of instruction. The pre-experimental research design was employed with a definitive instructional guide as the final output of the study. The subjects of the study consist of one intact class of Bachelor of Secondary Education Major in Mathematics who are enrolled in Math 213c (Solid Geometry) during the first semester of the school year 2015-2016 purposively selected for the study. The instructional guide contains instructional plans which include worksheets, activity sheets, and homework sheets integrating students' spatial visualization tasks. All materials which were developed in the study were subjected to face and content validation through several subject area specialists. The findings reveal that there are remarkable changes for both the students' level of geometric thinking and spatial visualization ability. It can be concluded that the integration of spatial visualization tasks is effective not only in improving students' spatial visualization ability but also effective in assisting students raised their van Hiele level and preventing them drop their van Hiele level.

Keywords: patial visualization; geometric thinking; van Hiele Model; definitive guide; Solid Geometry

*Corresponding author.

Recibido: 18/02/2019 Aceptado: 02/06/2019

RESUMEN

Los alumnos que tienen la capacidad de manipular las formas en situaciones de juego libre, como construir, resolver problemas espaciales, dibujar objetos bidimensionales y tridimensionales, explorar las formas mediante acciones físicas, describir las formas desde diferentes perspectivas y encajar las formas, suelen tener un nivel más avanzado de pensamiento geométrico. Desde esta perspectiva, el nivel de pensamiento geométrico de los estudiantes está asociado a la capacidad de visualización espacial. Por lo tanto, el estudio se realizó específicamente para desarrollar una guía definitiva que integrara tareas de visualización espacial para mejorar el nivel de pensamiento geométrico de los estudiantes siguiendo el modelo de instrucción de van Hiele. Se empleó el diseño de investigación preexperimental con una guía de instrucción definitiva como resultado final del estudio. Los sujetos del estudio consisten en una clase intacta de la Licenciatura en Educación Secundaria con especialización en Matemáticas que están inscritos en Matemáticas 213c (Geometría Sólida) durante el primer semestre del año escolar 2015-2016 seleccionados intencionalmente para el estudio. La guía didáctica contiene planes de instrucción que incluyen hojas de trabajo, hojas de actividades y hojas de tareas que integran las tareas de visualización espacial de los estudiantes. Todos los materiales que se desarrollaron en el estudio fueron sometidos a una validación facial y de contenido a través de varios especialistas en la materia. Los resultados revelan que hay cambios notables tanto en el nivel de pensamiento geométrico como en la capacidad de visualización espacial de los alumnos. Se puede concluir que la integración de las tareas de visualización espacial es eficaz no sólo para mejorar la capacidad de visualización espacial de los estudiantes, sino también para ayudar a los estudiantes a elevar su nivel de van Hiele y evitar que bajen su nivel de van Hiele.

Palabras clave: visualización patial; pensamiento geométrico; modelo van Hiele; guia definitiva Geometria solida

RESUMO

Os estudantes que têm a capacidade de manipular formas em situações de jogo livre, como construir, resolver problemas espaciais, desenhar objetos tridimensionais e tridimensionais, explorar formas através de ações físicas, descrever formas de diferentes perspectivas e ajustar formas são comumente observados para ter um nível avançado de pensamento geométrico. A partir dessa perspectiva, o nível de pensamento geométrico do aluno está associado à capacidade de visualização espacial. Assim, o estudo foi conduzido especificamente para desenvolver um guia definitivo integrando tarefas de visualização espacial para melhorar o nível de pensamento geométrico dos alunos seguindo o modelo de instrução de van Hiele. Projeto de pesquisa pré-experimental foi empregado com guia instrutivo definitivo como o resultado final do estudo. Os sujeitos do estudo consistem em uma classe intacta de Bacharel em Educação Secundária Maior em Matemática que estão matriculados na Matemática 213c (Geometria Sólida) durante o primeiro semestre do ano letivo de 2015-2016 propositadamente selecionados para o estudo. O guia de instrução contém planos de instrução que incluem planilhas, folhas de atividades e folhas de trabalhos de casa que integram as tarefas de visualização espacial dos alunos. Todos os materiais que foram desenvolvidos no estudo foram submetidos a validação de rosto e conteúdo através de vários especialistas da área. As descobertas revelam que há mudanças notáveis tanto no nível de pensamento geométrico dos alunos quanto na capacidade de visualização espacial. Pode-se concluir que a integração das tarefas de visualização espacial é eficaz não só para melhorar a capacidade de visualização espacial dos alunos, mas também para ajudar os alunos a elevar o nível de van Hiele e impedir que abandonem o nível de van Hiele.

Palavras-chave: visualização patial; pensamento geométrico; Modelo de van Hiele; guia definitivo; Geometria sólida

I. INTRODUCTION

We live in a three-dimensional world, thus, knowing how to describe it with a certain level of conciseness is a necessary skill that one must have. According to the National Council of Teachers of Mathematics (NCTM), our appreciation of our three-dimensional world requires knowledge in Geometry. Through this course, learners are able to better identify, describe and compare objects, such they can classify, model, or draw these objects in two or three dimensional presentations. In doing so, they develop visual and spatial skills, enabling them appreciation of Geometry as a necessary tool to describe and model the objects as presented by the world around them; which, in turn allows them to analyze and recognize relationships between and among Geometric figures, their congruence and similarity.

However, many studies have pointed out teaching and learning difficulties in Geometry (Salau,M. O, 1995). Factors for the mediocre performance of students in the subject include variables related to curriculum, teachers, students, home and learning materials. (Amazigbo 2000) stated weak foundational knowledge, specifically in primary school Mathematics, lack of incentives for teachers, unqualified teachers in the system, lack of learner's interest, perception of difficulty in the subject, class size and psychological or imaginary fear of the subject are some of the factors that bolster poor performance of students in the subject. As such, curriculum changes in Mathematics are undertaken as necessary to improve school Mathematics teaching to meet the ever changing needs of society, science and technology.

To support the quality of instruction in Geometry, Pierre Van Hiele and Dina Van Hiele-Geldof developed a Geometry learning model, which was designed to help identify the students' Geometric level of thinking, such that instruction is designed for their particular level, in order to aid them in moving to the next higher level. Van Hiele model features five distinct levels of Geometric thinking: recognition, analysis, informal deduction (order), formal deduction, and rigor. In this model, instruction follows a scheduled sequence according to the students' level , to progress them to a higher level of thinking. The instruction delivery follows a phased sequence designed after the levels of thinking as identified in the model. These five phases are: **inquiry/ information,** discussion between teacher and student concerning a Geometric topic; **guided orientation**, exploring the properties of figures by experimentation; **explication**, forming a network of relations regarding the Geometric topics; **free orientation**, challenging the students independently; and **integration,** incorporate students' knowledge about specific topic.

The model was proven effective as evidenced by numerous researchers (Aydin, Nuh, 2009); (Yazdani, Mohammad, 2001); (Simard, Carole, 2007); (Meng, Chew Cheng, 2009); (Unal, Hasan, 2009).

Students who have the ability to manipulate shapes in free play situations, such as building, solving spatial problems, drawing two and three dimensional objects, exploring shapes through physical actions, describing shapes from different perspectives and fitting shapes together are commonly observed to have a more advanced level of Geometric thought. Similarly, students who actively engage themselves in various planned activities in Geometry have an apparent developed Geometric thought.

The foregoing discussion prompted the researchers to conduct a study specifically to develop a definitive guide integrating spatial visualization tasks to enhance students' level of Geometric thinking following the Van Hiele model of instruction.

2. METHOD

The pre-experimental method of research was employed specifically, one group pre-test/post-tests design.

Pre-experimental method is a type of experimental design which does not include a control group. A single group of participants is studied, so that there is no comparison between a treatment group and a control group. This group is pretested using an instrument before they are exposed to the treatment, and then post-tested using the same instrument at the end of the treatment.

The subjects purposively selected for the study are members of the class of Bachelor of Secondary Education Major in Mathematics who are enrolled in Solid Geometry (Math 213c) during the first semester of school year 2015-2016.

The researcher used a Geometry test called **Van Hiele Geometry Test (VHGT)** designed by Usiskin (1982) to measure the students' level of Geometric thought. The VHGT was administered to the participants by the researcher during a single class period. The test consists of 25 multiple-choice Geometry questions.

In this study, the 1-5 scheme was used to identify the students' levels of Geometric thought labeled: Level 1 – Visualization; Level 2 – Analysis; Level 3 – Informal Deduction; Level 4 – Formal Deduction; and Level 5 – Rigor. This scheme allows the researcher to use Level 0 for students who do not function at what the Van Hiele's named the ground or basic level, visualization. This scheme is also consistent with Pierre Van Hiele's numbering of the levels. All participants' answer sheets from VHGT were read and scored by the researcher.

Students at **Level 0 (Pre- recognition level)** perceive Geometric shapes, but attend to only a subset of a shape's visual characteristic (Clements DH, Battista M.,., 1990). They are unable to identify many common shapes and they view Geometric concepts as total entities rather than having components or attributes. Geometric figures are recognized by their shape as a whole, that is, by their physical appearance, not by their parts or properties. Students can name a Geometric shape by its appearance alone. They can identify, name and compare Geometric shapes in their visible form (Fuys D., 2012). The properties of a figure play no explicit role in the identification process (Pegg J, Davey G., 1998). A person functioning at this level can learn Geometric vocabulary, can identify specified shapes, and given a figure, can reproduce it (Crowley, Mary L., 1987). At this level, students can associate certain objects such as faces, edges, vertices without necessarily taking note of the angles, sizes, edge lengths, parallelism, etc. The presence of these Mathematical elements in students' answers confirms their visualization level (Gutierrez, Angel, 1992)

Learners at **Level 2 ((Analysis or Descriptive Level)** identify a figure by its properties, which are seen as independent of one another (Pegg J, Davey G., 1998). Through observation and analysis, they are able to identify the attributes of the figures, discover properties and rules (Malloy C., 2002). Generally, they can distinguish and identify the properties of Geometric figures, however they do not yet understand the difference between these properties and between different figures (Van Hiele, P. M., 1986).

Informal Deduction refers to the third level of geometric thinking where the learners discover and formulate generalizations about previously learned properties and rules and develop informal arguments to justify those generalizations (Malloy C., 2002). Students no longer perceive figures as consisting of a collection of discrete, unrelated properties. Rather, they now recognize that one property of a shape proceeds from another. They also understand relationship between different figures (Pegg J, Davey G., 1998).

Formal Deduction refers to the fourth level of geometric thinking where the learners prove theorems deductively and understand the structure of the geometric system (Malloy C., 2002), understand necessary and sufficient conditions and can develop proofs rather than relying on rote learning, and can construct their own definitions of shapes (Pegg J, Davey G., 1998).

Rigor refers to the fifth level of geometric thinking where the learners establish theorems in different systems of postulates and can compare and analyze deductive systems (Malloy C., 2002). By using the classical, modified and forced Van Hiele levels for classification, almost every student can be assigned to a Geometric level. If almost all the students have a Van Hiele level, the data become easier for analysis. The weighted sum score for the classical Van Hiele level, the modified Van Hiele level and the forced Van Hiele level are adopted from Usiskin (1982).

In the following paragraph, some examples are presented in order to explain how to classify student's Van Hiele level. Usiskin (1982) has mentioned that the grading method carried out to classify students' Van Hiele level (the classical and modified Van Hiele level) not only requires students at that specific level but also at all preceding levels. Thus, for instance, when a student who satisfies the criterion of Van Hiele level 1, level 2 and level 3, he or she gets 7 points (1 + 2 + 4) and may be assigned to the classical Van Hiele level 3 (see Table 1 below).

Table 1. The weighted sum score for classical van hiele level

Level	**Weighted sum score**
0	0
1	1
2	3
3	7
4	15
5	31
No fit	Other Score

Besides, if a student who only satisfies the criterion of level 1, level 2, level 3 and level 5, he or she gets 23 points (1+ 2 + 4 + 16) and may not be assigned to any classical Van Hiele level (because the student does not satisfy all the preceding levels of level 5). However, the student may be assigned to the modified Van Hiele level 3 since he or she satisfies the criterion of level 1, level 2 and level 3 continuously (see Table 2).

Table 2. The weighted sum score for modified van hiele level

Level	***Weighted sum score***
0	0 or 16
1	1 or 17
2	3 or 19
3	7 or 23
4	15 or 31
No fit	Other Score

Students who get other points that are not included in Table 2 and Table 3 may be described as "no fit" in classical and modified Van Hiele level. On the other hand, if a student cannot be assigned also to a modified Van Hiele level, a forced Van Hiele level may be used to determine students' Geometric level. According to Usiskin (1982), students are assigned to a forced Van Hiele level n if "(a) the students meets the criterion at levels n and n-1 but perhaps not at one of n-2 or n-3, or (b) the student meets the criterion at level n, all levels below n, but not at level n+1 yet also meets the criterion at one higher level".

For instance, if a student who only satisfies the criterion of level 2, level 3 and level 4, he or she gets 14 points (2 + 4 +8) and may not be assigned in the classical and modified Van Hiele level but in the forced Van Hiele level 4 (see Table 3).

Table 3. the weighted sum score for forced van hiele levels

Level	Corresponding Weighted Sum
0	0 2 4 8 16 18 20 24
1	1 5 9 17 21 25
2	3 11 19 27
3	6 7 22 23
4	13 14 15 29 30 31

The respondents were also subjected to the Purdue Visualization of Rotations Test (ROT) to determine the spatial visualization ability levels of students. The examination is one element of the Purdue Spatial Visualization Test Battery (Guay, 1977). The test consists of 20 items that have students to: (1) study how the object in the top of the line of the question is rotated; (2) visualize what the object shown in the middle of the line of the question looks like when rotated in exactly the same manner, and (3) select from among the five drawings (A, B, C, D, and E) given in the bottom line of the question the one that looks like the object rotated in the correct position.

Carpenter and Just (1986) studied university students with low and high spatial ability, where student participants were given alphabet blocks to judge whether each could represent the same block (Carpenter, P. A., & Just, M. A., 1986) Analyzing the students' eye fixation, researchers of the said study found that low spatial ability participants sometimes had to turn a particular block more than once, while high spatial ability participants rarely had to turn the same block more than once. They also analyzed the students' performance using the Shepard-Metzler Mental Rotation Task, and found low performance students tend to rotate the figures at a slower rate than those performing well. They also observed that the low spatial group had more difficulty and that tend to restart various processes because they have difficulties remembering or recalling intermediate products. These observations led Carpenter and Just to conclude that "low spatial subjects have difficulty maintaining a spatial representation while performing transformations".

Through the findings of the study, the researchers designed an instructional guide containing instructional plans that include worksheets, activity sheets, homework sheets and other tasks which enhance students' spatial visualization ability. All materials which were developed in this study were subjected to face and content validation through several subject area specialists.

III. RESULTS AND DISCUSSION

A. Entry Competence of the Students

Levels of Geometric Thinking. The students were assigned to the Van Hiele levels using a grading method which was based on the "4 of 5 correct" success- criterion as suggested by Usiskin (1982). By this criterion, they are believed to have mastered a certain level when they answer at least 4 out of the 5 items in any of the 5 subtests within the test.

Table 4 presents the students' levels of Geometric thinking before their exposure to the Van Hiele Instruction Model. As shown in the table, 12 out of 35 respondents or 34.29% were functioning at **Level 0**, and 17 out of 35 or 48.57% were functioning at **Level 1** prior to the instruction. Only one (1) out of 35 or 2.86% of the respondents was functioning **Level 4**. It can be noted that none of the respondents operate at **Level 5**.

The mean level of 0.89 suggests that students were situated between **Level 0** and **Level 1** prior to the Van Hiele Model instruction.

Table 4. entry competence of the students in terms of their levels of geometric thinking

Level	Description	Frequency (f)	Percent (%)	Mean/ Standard Deviation
Level 0	Pre-recognition	12	34.29	0.89/0.87
Level 1	Visualization	17	48.57	
Level 2	Analysis	5	14.29	
Level 3	Informal Deduction	0	0.00	
Level 4	Formal Deduction	1	2.86	
Level 5	Rigor	0	0.00	
TOTAL		**35**	**100.00**	

Spatial Visualization Ability. As shown in table 5, a mean of 7.94 suggests that **students' spatial visualization ability** is **low** before the intervention. A standard deviation of 3.20 represents the degree of clustering of the scores from the mean. This implies that nearly sixty-eight percent (68%) or twenty-four (24) respondents registered scores between 4.74 and 11.14.

Table 5 likewise presents the distribution of spatial visualization ability of the respondents before the treatment. The figure shows that none of the respondents have **high or very high spatial visualization ability**. Five (5) out of thirty-five (or 14.29%) of the respondents had **very low spatial visualization ability**.

Table 5. entry competence of the students in terms of their levels of spatial visualization ability

Score	Description	Frequency (f)	Percent (%)	Mean/ Standard Deviation
19-20	Very High Spatial Visualization Ability	0	0.00	7.94/ 3.20 (Low Spatial Visualization Ability)
16-18	High Spatial Visualization Ability	0	0.00	
10-15	Average	13	37.14	
5-9	Low Spatial Visualization Ability	17	48.57	
0-4	Very Low Spatial Visualization Ability	5	14.29	
TOTAL		**35**	**100.00**	

Brown and Wheatley (1990), found that it is possible for students with **low spatial ability** to earn average or above average performance in standardized tests yet still be unable to solve non-routine problems (D.L. Brown and G.H. Wheatley, 1990). Average or above average performance in standardized test does not always indicate good analytical thinking in Mathematical tasks as a whole. Further, in the same study, they suggested that it is possible for **high spatial ability** students to perform average or below average in standardized tests but show an advanced understanding of Mathematical ideas, and be able to solve creatively non-routine Mathematical problems. These findings again highlighted the importance of spatial ability as an important factor for conceptual understanding and giving meaning to a Mathematical activity.

Further, in the study conducted by Unal (2008), those pre-service teachers with high or mid-range spatial ability scores showed both a greater beginning level of Geometric understanding and a greater improvement in Geometric understanding than the student with the lowest spatial ability score (Unal, Hasan, 2009).

B. Students' Level of Geometric Thought and their Spatial Ability after the Exposure to the Van Hiele Model

After eight weeks of instruction following the Van Hiele Model, students were subjected to the same assessment using the same instruments: Van Hiele Geometry Test for the levels of Geometric Thinking and Purdue Visualization of Rotation Test for the Spatial Visualization Ability. Changes in the students' level of Geometric thought and their spatial ability were examined and discussed accordingly.

Levels of Geometric Thinking. To assess the students' Van Hiele levels after their exposure to the Van Hiele Model, the researchers also used the criterion for success considered during the pre- test.

Table 6 reveals that after the intervention: 3 respondents or about 8% were classified at **Level 5**; 2 respondents or about 6 % were classified at **Level 4**; 6 respondents or 17% were classified at **Level 3**; 7 respondents or 20 % were classified at **Level 2**; 10 respondents or about 29% were classified at **Level 1**; and 7 respondents or about 20 % were classified at **Level 0**. These data implies that after instruction, majority of the students moved to a higher Van Hiele level.

A mean score of 1.86 suggests that the respondents were functioning between **Level 1 and level 2** after their exposure to the Van Hiele Model. Furthermore, a standard deviation of 1.53 implies the degree of variability of the scores about the mean.

Table 6. students' levels of geometric thinking after their exposure to the van hiele model

Level	Description	Frequency (f)	Percent (%)	Mean/ Standard Deviation
Level 0	Pre-recognition	7	20.00	1.86/1.53
Level 1	Visualization	10	28.57	
Level 2	Analysis	7	20.00	
Level 3	Informal Deduction	6	17.14	
Level 4	Formal Deduction	2	5.71	
Level 5	Rigor	3	8.57	
TOTAL		**35**	**100.00**	

Spatial Visualization Ability. After the students' exposure to the Van Hiele Model, their spatial visualization ability was determined following the scoring guide and procedure as indicated in the ROT. The results are presented in table 7.

As reflected in table 7, a mean score of 12.74 suggests that the students have **average spatial visualization ability** after their exposure to the Van Hiele Model. The standard deviation of 3.89 implies the degree of clustering of the scores about the mean. This means that nearly 68% or 24 of the respondents obtained scores between 8.85 and 16.63. Moreover, the table presents the distribution of the spatial visualization ability of the respondents after the intervention

Table 7. students' spatial visualization ability after their exposure to the van hiele model

Score	Description	Frequency (f)	Percent (%)	Mean/ Standard Deviation
19-20	Very High Spatial Visualization Ability	0	0.00	12.74/ 3.89 (Average Spatial Visualization Ability)
16-18	High Spatial Visualization Ability	9	25.71	
10-15	Average	21	60.00	
5-9	Low Spatial Visualization Ability	4	11.43	
0-4	Very Low Spatial Visualization Ability	1	2.86	
TOTAL		**35**	**100.00**	

The table shows that 9 out of 35 respondents or 25.71% registered **high spatial visualization ability** after their exposure to the Van Hiele Model instruction, while only one (1) of them registered very low spatial ability after the treatment.

Percentage Increase of the Levels of Geometric Thinking and Spatial Visualization Ability. To further examine the changes that took place in the students' level of Geometric thinking and their spatial visualization ability after their exposure to the Van Hiele Model, the results of the pre- test and post- test were compared noting the percentage increase for both the levels of Geometric Thinking and Spatial Visualization Ability.

Table 8 presents the percentage increase in the students' level of Geometric thinking and spatial visualization ability after their exposure to the Van Hiele Model instruction. As reflected in the table, students' level of Geometric thinking registered a percentage increase of 108.99% while their spatial visualization ability registered a percentage increase of 60.45%. It can be noted that there are remarkable changes for both the students' levels of Geometric thought and spatial ability.

Table 8. Percentage increase of the levels of geometric thinking and spatial visualization ability

Dimensions	Entry Competence	After the Exposure to the Van Hiele Model	Increase in Percent (%)
Levels of Geometric Thinking	0.89	1.86	108.99
Spatial Visualization Ability	7.94	12.74	60.45

The said findings are significant based on the computed t-value of -3.978 for levels of Geometric thinking and -11.779 for spatial visualization ability - results shown in table 9. This means that there are significant differences between the students' level of Geometric thinking and their spatial visualization ability before and after their exposure to the Van Hiele model instruction.

Table 9. test for the significant difference between the students' levels of geometric thinking and spatial visualization ability before and after their exposure to the van hiele model

Dimensions		Mean	Standard Deviation	Degrees of freedom	Computed t-value
Levels of Geometric Thinking	Before	0.89	0.87	34	-3.978**
	After	1.86	1.53		
Spatial Visualization Ability	Before	7.94	3.20	34	-11.779**
	After	12.74	3.89		

**highly significant

The results supports the findings of Liu (2005) who concluded that the use of the Van Hiele-based instruction is not only effective in assisting students raised in Van Hiele level but also in preventing students drop their Van Hiele level [18].

Malloy (2002) observed that the quality and nature of the experience in the teaching and learning program ought to influence the advancement from a lower to a higher level of Geometric thought. Moreover, Chan (2006) noted effective learning under Van Hiele Theory takes place when students are cognizant of their study objectives and are made aware when these objectives are achieved, and when they are given opportunities to reflect and discuss about their learning which results in their movement to a higher level of Geometric thought. Chan also contends that lecture and memorization as main methods of instruction will not lead to effective learning.

The results of this study are likewise consistent with the findings of Battista et al.(1982) who found that learners significantly improved their level of spatial ability after Van Hiele instruction. Basttista et al., using ROT, found that the control group who received spatial activities such as paper folding, tracing and symmetry, scored higher compared to a non-control group. While these improvements are greatly attributed to the specific activities in the instruction, the present study integrated and emphasized such activities in a Van Hiele based instruction.

IV. CONCLUSIONS AND RECOMMENDATIONS

Conclusions

In the light of the findings of the study, the following conclusions were drawn:

1. There is a significant difference between students' levels of Geometric thinking and their spatial visualization ability before and after their exposure to the Van Hiele model instruction.

2. The integration of spatial visualization tasks is effective not only in improving students' spatial visualization ability but also effective in assisting students raised their Van Hiele level and preventing them drop their Van Hiele level.

Recommendations

Based on the findings and conclusions of this study, the researchers devised following recommendations for consideration:

1. Since there are evidences which shows the effectiveness of the instructional guide developed to improve both the students' level of Geometric thinking and their spatial visualization ability, this can be used by teachers handling Geometry subjects

2. Geometry teachers are encouraged to take some professional training course that are related to Van Hiele model which provides not only an opportunity for them to improve their insight of Geometry but also help them understand the Van Hiele model which can directly influence their way of pedagogy in Geometry.

3. Geometry teachers should introduce in their instructional materials and methods, strategies in planning and delivering lessons that make use of the Van Hiele instruction.

4. Teacher education programs should incorporate the underlying principles of Van Hiele Model into courses dealing with instructional methods and curriculum design.

5. Writers of Geometry textbooks should base the content on particular features and characteristics of the Van Hiele model.

BIBLIOGRAPHIC REFERENCES

Amazigo, J.C. (2020). "Mathematics phobia diagnosis and prescription". National Mathematics centre first annual lecture, Abuja.

Aydin, N., et.al., (2009). "The Impact of Undergraduate Mathematics Courses on College Students' Geometric Reasoning Stages". The Montana Mathematics Enthusiast, vol. 6, nos 1 and 2.

Brown, D.L. and Wheatley, G.H. (1990). The Role of Imagery in Mathematics Reasoning, Proceedings of the 14th Annual Meeting International Group for Psychology of Mathematics Education Conference, Vol. 1, Mexico.

Carpenter, P. A., & Just, M. A. (1986). Spatial ability: An information processing approach to psychometrics. In R. J. Sternberg (Ed.), Advances in the psychology of human intelligence, Vol. 3. Hillsdale, NJ: Erlbaum

Clements, D.H., Battista, M. (1990). The effects of logo on children's conceptualization of angle and polygons. Journal for Research in Mathematics Education

Crowley, M. L. (1987). "The Van Hiele Model of the Development of Geometric Thought" In Learning and Teaching Geometry, K- 12, 1987 Yearbook of the National Council of Teachers of Mathematics.

Fuys, D., et.al., (2012). The Van Hiele model of thinking in Geometry among adolescents. Journal for Research in Mathematics Education: Monograph No. 3, 1988 In Alex, et.e. "A Survey of South African Grade 10 Learners' Geometric Thinking Levels in Terms of the Van Hiele Theory.

Gutierrez, A. (1992). "Exploring the Links between Van Hiele Levels and 3- Dimensional Geometry".

Liu, K. (2005). "The effectiveness of Van Hiele-based instruction", University of Hongkong.

Malloy, C. (2002). The Van Hiele Framework in Navigating Through Geometry in Grades 6 -8: NCTM.

Meng, C. (2009). "Pre-Service Secondary Mathematics Teachers' Geometric Thinking and Course Grade", Universiti Sains Malaysia.

Pegg J, & Davey G. (1998). Interpreting student understanding in Geometry: A synthesis of two models. In: R Lehrer, D Chazan (Eds.): Designing Learning Environments for Developing Understanding of Geometry and Space. New Jersey: Lawrence Erlbaum Associates, Publishers.

Salau, M. O. (1995). "Analysis of students enrollment and performance in Mathematics at the senior secondary certificate level". Journal of curriculum studies.

Simard, C., et.al. (2007). "Exploring the Van Hiele Levels of Prospective Mathematics Teachers".

Unal, H., et. al., (2009). "Differences in Learning Geometry among High and Low Spatial Ability Pre- service Mathematics Teachers". International Journal of Mathematics Education in Science and Technology, Vol. 40, No. 8.

Van Hiele, P. M., (1986). Structure and insight: A theory of mathematics education.

Yazdani, M. A. (2001). "Correlation between Students' Level of Understanding according to the Van Hieles' Model and Students' Achievement in Plane Geometry". Journal of Mathematical Sciences and Mathematics Education.